IDS 101 Doing Mathematics

Today

1. Sleeping, studying, eating, and self-care (see Google doc)
   (a) Avoid all-nighters.
   (b) Set a regular sleep schedule.
   (c) Use your freedom responsibly.
   (d) Take care of yourself! Classes are important, but not as important as you.
2. Questions about anything as the term gets underway?
3. Report on last time's problems.
4. Graphs! (See below)

Problems

1. **Definition:** A graph is a set of elements called **vertices** (singular: **vertex**) and a list of pairs of vertices called **edges**. We often represent a graph by making each vertex a point and drawing a (not necessarily straight) line between vertices to indicate edges. If two vertices belong to an edge, we say they are **adjacent**; a vertex and an edge to which it belongs are **incident**. If u and v are adjacent vertices, we usually denote the edge between them by uv. The **degree** of a vertex is the number of times it appears in the list of pairs (i.e., the number of edges incident with it).
2. **Definition:** A **path** in a graph is an alternating sequence of vertices and edges of the form $v_1, v_1v_2, v_2, v_2v_3, v_3, \ldots, v_{n-1}, v_{n-1}v_n, v_n$ in which no vertex is repeated and each edge joins the vertices listed immediately before and after it. We usually omit the edges when describing a path in G, taking them as understood to be the edges joining consecutive vertices. A **cycle** in a graph is a path $v_1, \ldots, v_n$ in the graph along with one additional edge from v_n back to v_1.
3. **Notation:** If G is a graph, we denote its set of vertices by $V(G)$ and its list of edges by $E(G)$.
4. Special graphs:

(a) The **complete** graph K_n is the graph with n vertices and an edge between every pair of distinct vertices.

(b) The **path** graph P_n is a graph that consists only of a path on n vertices.

(c) The **cycle** graph C_n is a graph that consists only of a cycle on n vertices.

(d) A **bipartite** graph is a graph G whose vertices can be partitioned into two sets in such a way that there are no edges between vertices in the same set. The sets are usually referred to as **sides**.

(e) The **complete bipartite** graph $K_{m,n}$ is a bipartite graph with m vertices on one side and n on the other and that has every possible edge from one side to the other.

(f)

5. **Definition:** A graph G is **connected** if every pair of vertices has some path between them. A **tree** is a connected graph with no cycles. If G is a tree, a vertex $v \in V(G)$ is a **leaf** if its degree is 1.

6. **Definition:** A graph G is **planar** is there is some way to draw G so that none of its edges cross. A **plane drawing** of a graph is a drawing in which no edges cross. (Note that planar refers to potential, while a plane drawing is a realization of that potential.)

7. A **subgraph** H of a graph G with $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$.

8. A **proper coloring** of a graph G is an assignment of colors to its vertices in such a way that adjacent vertices receive different colors. The **chromatic number** of a graph G is the smallest number of colors for which a proper coloring exists, denoted $\chi(G)$.

Homework:

1. Explore these concepts. Draw examples, try to find some chromatic numbers, make conjectures, etc.

2. Visit all of your other professors in their offices this week.